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**Annex 1 to**

***D4.3.4 Calculation of contact stress***

# Hertzian solution vs. non-Hertzian solution

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Often for wheel-rail contact Hertzian contact is presumed. That means the contact pressure and the contact area are calculated according to the Hertz theory. Hertz theory is based on the assumption that the contact surfaces of the two contacting bodies are perfectly smooth so that they can be expressed by quadratic functions. As a result the contact geometry is characterized by four principal radii of curvature at the contact point of the wheel and rail.

For WP4.3 a non-Hertzian method is proposed for the solution of contact stress. This is because that head checks usually occur at gage shoulder and gage corner, and the contact is often conformal (This is evidenced, e.g. by the results of the VAS WET #1 test). The contact area may be broad and across the gage shoulder down the gage face, where the radius of curvature of the contact surfaces varies largely. Hence the contact may not be represented by the principal radii at any one point. Also two-point contact or even multiple point contact may play an important role for the formation of the conformal contact and for the analyses.

Below three examples are given to show the difference between Hertzian and non-Hertzian contact solutions.

**Example 1**: For the profile combination shown in figure 1, with the following parameters

Wheel radius = 460mm,
Gage = 1435mm,
Wheel back distance = 1353mm.
Wheel load = 10E5 N.
Modulus of rigidity (Shear modulus) G=0.82E11,
Poisson ration = 0.28
Rail inclination = 1/20
Pressure in figures is normalized with G.
Wheelset has no lateral displacement with respect to track center.

The contact area and pressure distribution of figure 2 are obtained. The contact point on the wheel is in the area of R300mm. On the rail it is around the transition of R300 and R80.

For the non-Hertzian solution, the maximum pressure is

Pz max =1.46 GPa, see fig 2(b).

For the Hertzian solution, there can be the following possibilities:

1. For the rail use the radius of R80 for the calculation →
   a. Pz max = 1.81 GPa,
   b. Semi-axies (a, b) = (3.2, 8.24)
2. For the rail use the radius of R300 for the calculation →
   a. Pz max = 0 GPa,
   b. Semi-axies (a, b) = (0, ∞) because the wheel R300 concave profile fits perfectly with the rail convex profile so that mathematically and according to Hertz theory one of the semi axis of the elliptical contact area is infinite!
3. Any other approximations to find a weighed radius of curvature so that solution can be continued with Hertzian method.

**Example 2:** The same conditions as for example 1, except for the rail inclination, which is now 0. See figure 3.

In this case the contact takes place between the R300 part of the wheel and R80 part of the rail so that for the non-Hertzian solution

Pz max = 1.9 GPa

While for the Hertzian solution

Pz max = 1.8GPa, with semi-axis a=3.2, b=8.24mm.

In this case the Hertzian and non-Hertzian solutions agree with each other. The small difference can mainly be attributed to the discretization in the non-Hertzian solution.

**Example 3**: This example is taken from fig. 4(b) and fig. 5(a) of annex *Analysis_of_VAS_WET_1_Test.pdf*. The rigid body contact point is at 28.8mm from the rail center (straight line distance). The contact covers the area 19mm from the vertical rail profile symmetry line till 33mm from the rail symmetry line. The contact area has a curvi-linear length of about 23 mm in the lateral direction. See figure 4.

Obviously in such a conformal contact of about 23mm long in the y (lateral) direction, the worn profile does not have a constant radius of curvature; hence Hertzian solution is virtually not applicable.

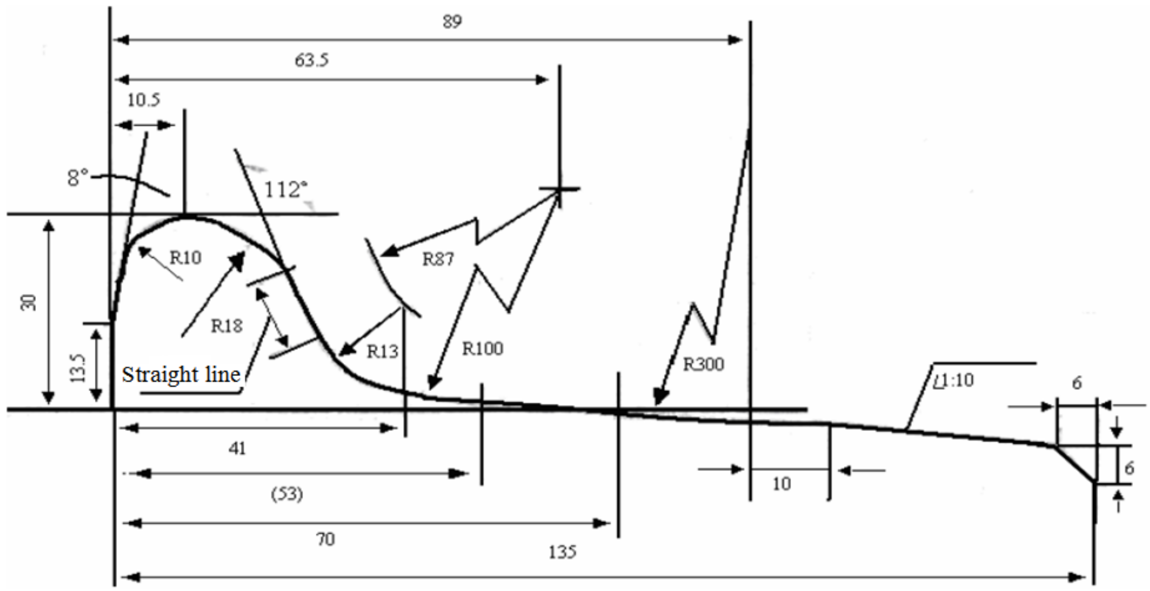


Figure 1(a) An analytically defined wheel profile

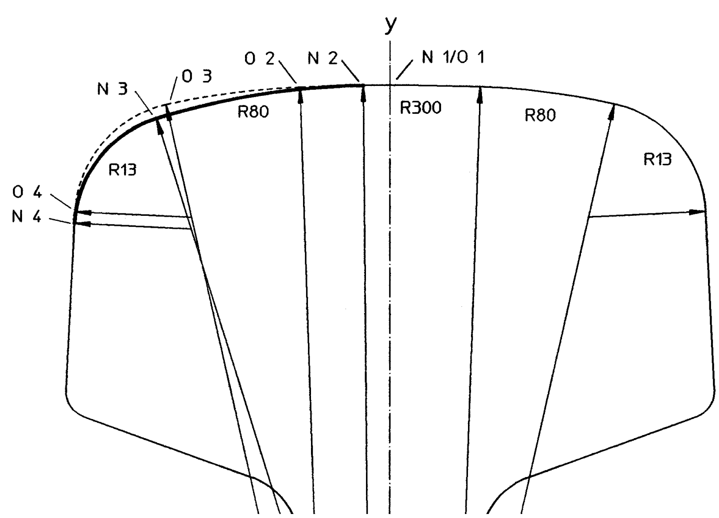


Figure 1(b) UIC54E1 profile (Use the dashed line for the left half)
Figure 1: the wheel and rail profiles used for example 1.

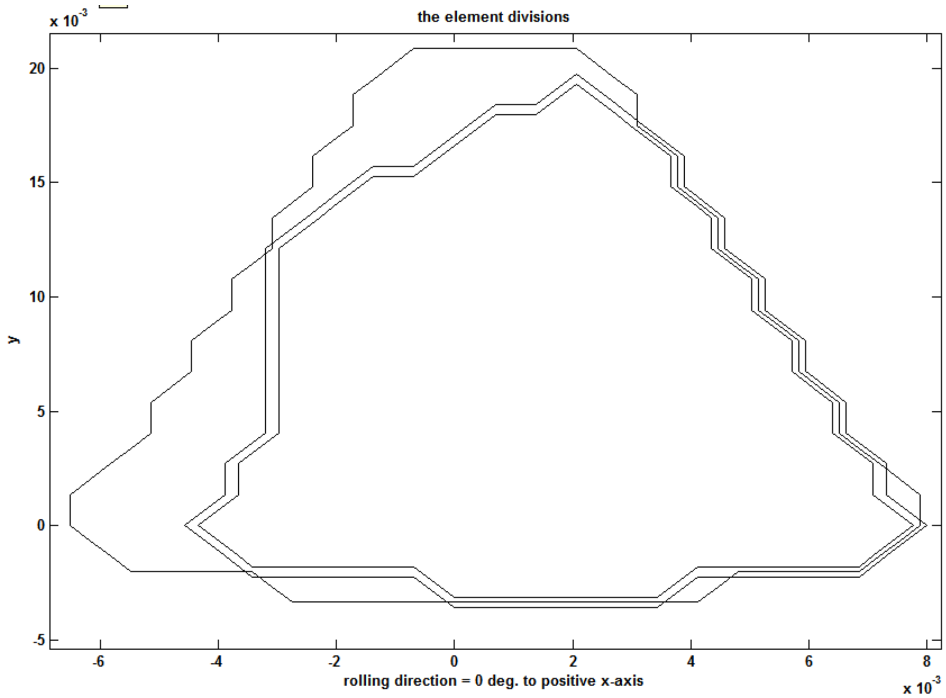


Figure 2(a): Contact area (Ignore the double line). Dimension is meter. Positive X-axis is in rolling direction, and Y-axis is in lateral direction. Positive Y-axis points from field side to gage side.

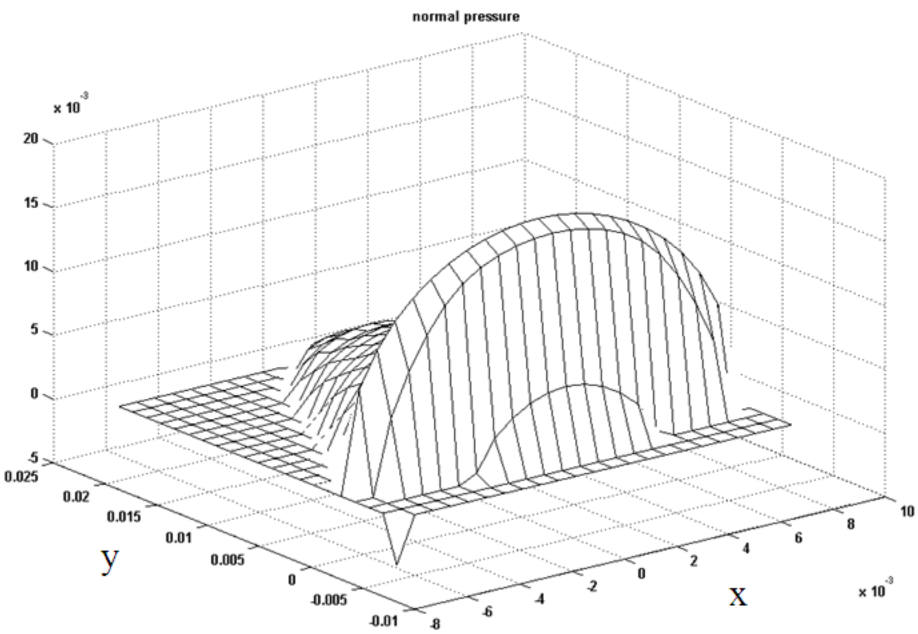


Figure 2(b): Contact pressure. Vertical axis is contact pressure, normalized by modulus of rigidity G = 0.82E11 Pa. Horizontal axes are the x and y directions of the contact area, dimension is meter.

Figure 2: Non-Hertzian solution of example 1

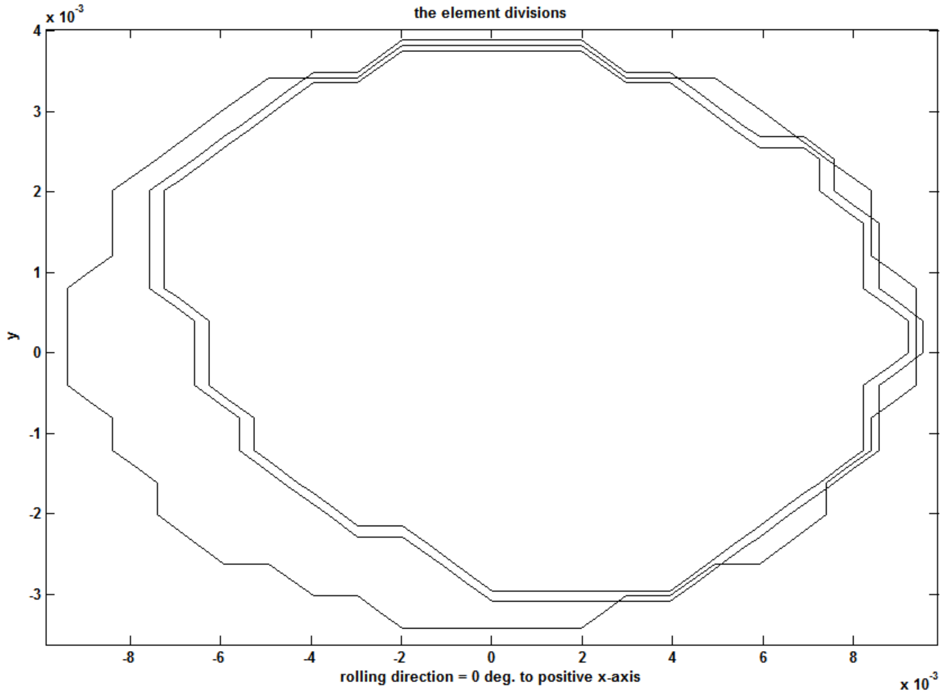


Figure 3(a): Contact area (Ignore the double line). Dimension is meter. Positive X-axis is in rolling direction, and Y-axis is in lateral direction. Positive Y-axis points from field side to gage side.

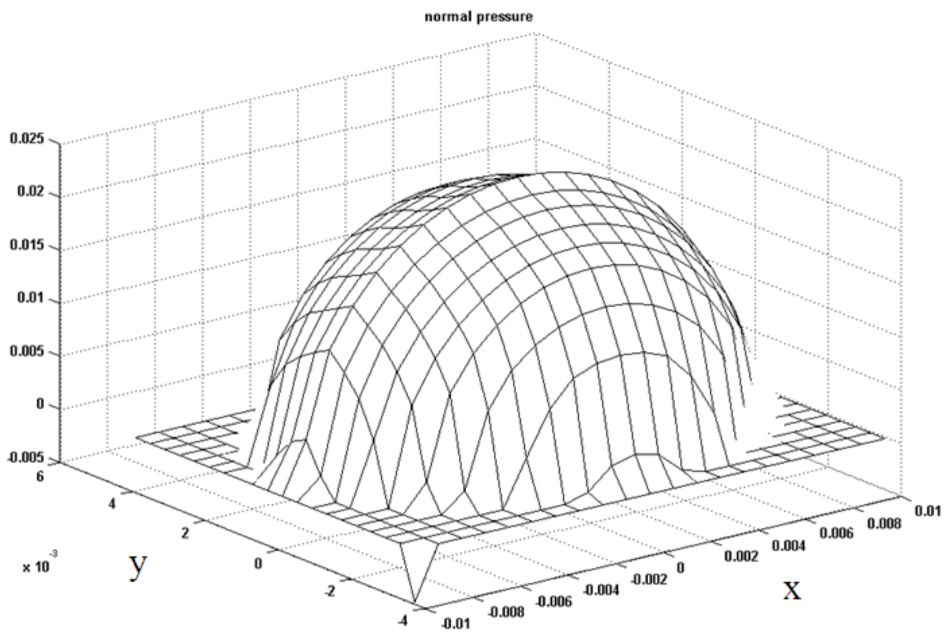


Figure 3(b): Contact pressure. Vertical axis is contact pressure, normalized by modulus of rigidity G = 0.82E11 Pa. Horizontal axes are the x and y directions of the contact area, dimension is meter.

Figure 3: Non-Hertzian solution for example 2

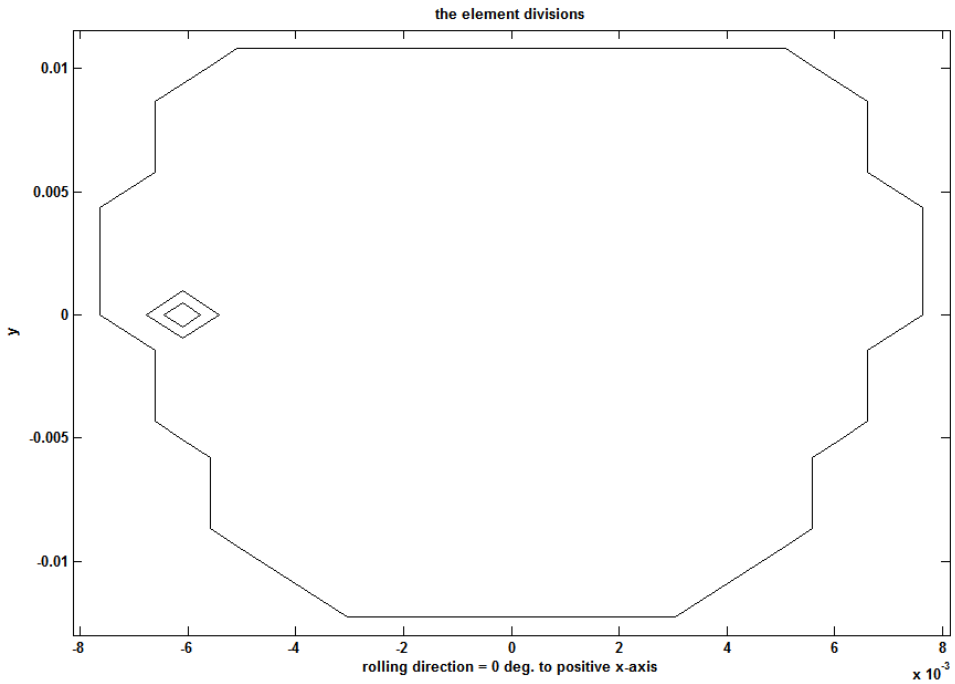


Figure 4(a): Contact area (Ignore the double line). Dimension is meter. Positive X-axis is in rolling direction, and Y-axis is in lateral direction. Positive Y-axis points from field side to gage side.

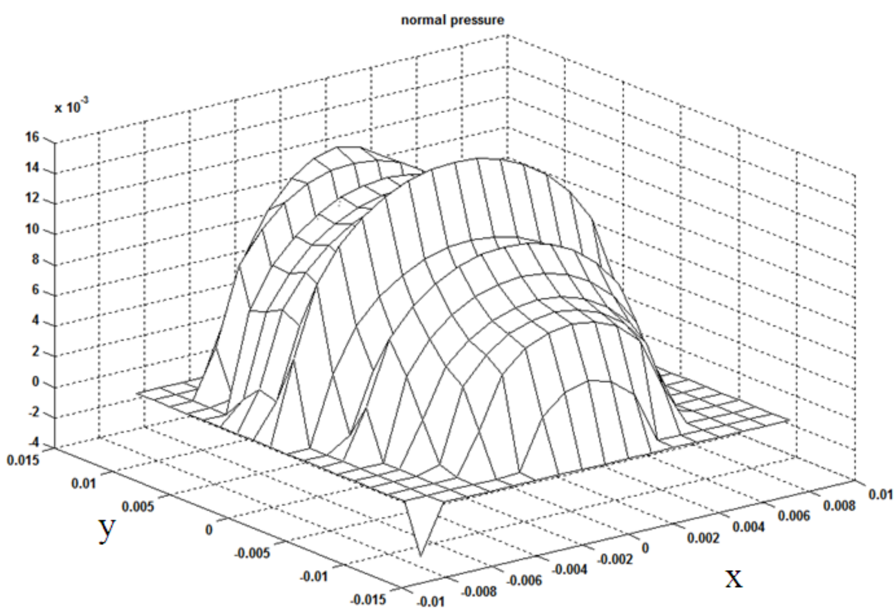


Figure 4(b): Contact pressure. Vertical axis is contact pressure, normalized by modulus of rigidity G = 0.82E11 Pa. Horizontal axes are the x and y directions of the contact area, dimension is meter.

Figure 4: a non-Hertzian solution taken from figure 5(a) of the annex 3 D4.3.4 repot, as example 2.